The Broad Sense Chain-Making and Chain-Coupling Theorems of Element Grid in 2-D Problems

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***Abstract*-The purpose of these papers is to answer a basic problem of finite element method that is how do the elements which have determined shapes form the element grid which has determined structure. In order to study this problem, a method called chain-making and chain-coupling is established. Through this method the quantitive relationships between the geometric structures of the element grid and the geometric structures of its elements are founded. This problem is studied both in 2-D and 3-D cases and here is the second part of 2-D case conclusions.**

I. INTRODUCTION

In order to answer the problem of how do the elements which have determined shapes form the element grid which has determined structure, a method called chain-making and chain-coupling is established. In the method, the relationships between elements in an element grid are classified according to their topological relationships. Then through defined steps the element grid which has complicated relationships between its elements can be disassembled to another element grid which just has a single kind of 0-dimension relationship between its elements. The disassembled element grid is called chain, and the disassemble procedure is called chain-making.

In a chain, elements have simple relationships of 0-dimension, so the quantitive relationships between the geometric structures of the chain and the geometric structures of its elements can be found relatively easily.

On the other hand, the 0-dimension relationships between the elements in a chain are inherited from the original element grid, so the original element grid can be reassembled from its chain. The procedure about how does the original element grid to be reassembled from its chain is called chain-coupling. By studying the chain-coupling procedure, basic properties about how do the elements in a chain form the original element grid are found, and the quantitive relationships between the geometric structures of the element grid and the geometric structures of its elements are found. So the problem of how do the elements which have determined shapes form the element grid which has determined structure are answered

II. NARROW SENSE CHAIN-MAKING AND CHAIN-COUPLING THEOREM OF 2-D GRID

Definition 2-1: Vertex Chain

On a set N which consists of elements with a number of n, a function *f* can be defined as following:

1. The first element e_1 just has one 0-dimension relationship with the second element e_2 through a common vertex v_{12};

2. The ith(1<i<n) element e_i has one 0-dimension relationship with the (i-1)th element e_{i-1} through a common vertex $v_{(i-1)i}$, and it has one 0-dimension relationship with the (i+1)th element $e_{(i+1)}$ through a common vertex $v_{i(i+1)}$;

3. The nth-element e_n just has one 0-dimension relationship with the (n-1)th element $e_{(n-1)}$ through a common vertex $v_{(n-1)n}$;

Thus the elements in set N are mapped to a vertex chain through function *f*.

Theorem 2-1: The first chain-making theorem in 2-D grid

In a 2-D triangle grid, if there are not cut-vertex, cut-edge and interior boundary in it, this triangle grid can be made into a vertex chain.

Theorem 2-2: The second chain-making theorem in 2-D grid

In a 2-D rectangle grid, if there are not cut-vertex, cut-edge and interior boundary in it, this rectangle grid can be made into a vertex chain.

Theorem 2-3: In a 2-D vertex chain, if the element number is n, the edge number of every element is m, then the total edge number in the chain is l_c=n×m and the total vertex number in the chain is v_c= (n×m)-(n-1).

Theorem 2-4: Narrow sense chain-coupling theorem in 2-D grid

In a narrow sense chain-makeable 2-D grid among the number of elements: n, the edges of every element: m, total number of edges of the grid: l, number of boundary edges: l_b, number of interior edges: l_i; total number of vertexes of the grid: v, number of interior vertexes: v_i, the following quantitive relationships hold:

$$l_i=(n\times m-l_b)/2 \quad (1)$$

$$l=(n\times m+l_b)/2 \quad (2)$$

$$v=l-(n-1) \quad (3)$$

$$v_i=l-(n-1)-l_b \quad (4)$$

The concern concepts and the demonstration of these theorems can be found in reference [1].

III. BROAD SENSE CHAIN-MAKING THEOREM

As shown in reference [1], the condition and procedure for narrow sense chain-making and chain-coupling are quite strict and complicated. At the same time, along with these discussions, a very important property of chain-making and chain-coupling can be found, that is when the chain-coupling property of the nth element on a vertex chain is studied, the continuity of the nth element in the (n-1)th order chain-making grid and nth order chain-coupling grid are required. But when this continuity is satisfied, it is not required that the original element grid could be made into a entire vertex chain, and there could be some branches in the making out vertex chain.

Definition 3-1: Branched Vertex Chain

A function f can be defined on a set N which consists of elements with a number of n, if it makes the ith($i \leqslant n$) element e_i in the jth($j \geqslant 1$) branch has one 0-dimension relationship with the (i-1)th element e_{i-1} through a common vertex $v_{(i-1),I}$, then the elements in set N are mapped to a branched vertex chain through function f.

Definition 3-2: Branching Element in a Branched Vertex Chain

If branches appear on an element of a branched vertex chain, this element is called a branching element of the branched vertex chain.

Theorem 3-1: Chain-making Theorem in 2-D Grid

A 2-D grid can be made into one branched vertex chain at least.

IV. Broad Sense Chain-coupling Theorem

Theorem 4-1: Theorem of Branched Chain-coupling

Branches in a branched vertex chain have no influence on the result of chain-coupling.

Theorem 4-2: In a 2-D branched vertex chain consist of elements of m edges and with element number of n, there are n element, $l_c=n\times m$ edges and $v_c=(n\times m)-(n-1)$ vertexes in it.

Theorem 4-3: If there are x class touch vertexes with order of $d_u(1\leqslant u\leqslant x)$ and number of $n_u(1\leqslant u\leqslant x)$ for each class, in a 2-D grid with boundary edge number of l_b, then number of boundary vertex in it is $v_b=l_b-\sum_{u=1}^{x} n_u\times d_u$.

Theorem 4-4: Chain coupling theorem in 2-D grid

In a 2-D grid if there are x class touch vertexes in it, among the number of elements: n, the sides of every element: m, the number of interior boundary i, the order of the x class touch point $d_u(1\leqslant u\leqslant x)$, the number of d_u order touch point: $n_u(1\leqslant u\leqslant x)$, total number of sides of the grid: l, number of boundary sides: l_b, number of interior sides: l_i; total number of vertexes of the grid: v, number of interior vertexes: v_i, the following quantitive relationships hold:

$$l_i=(n\times m-l_b)/2 \tag{1}$$

$$l=(n\times m+l_b)/2 \tag{2}$$

$$v=l-(n-1)-i \tag{3}$$

$$v_i=l-n-i+\sum_{u=1}^{x} n_u\times d_u -l_b+1 \tag{4}$$

The concern concepts and the demonstration of these theorems would be given in full paper.

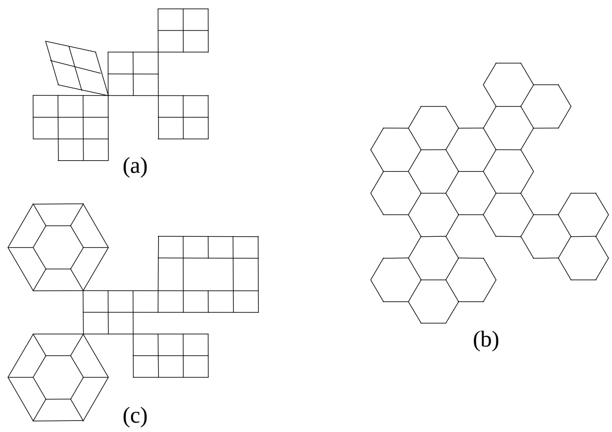

Fig.1. Sample grid in 2-D case

In fig.1 three sample grids are shown. In fig1(a) n=24, m=4, $l_c=96$, $v_c=73$, i=0, $\sum_{u=1}^{x} n_u\times d_u=4$, $l_b=44$, $l_i=26$, l=70, v=47, $v_i=7$. In fig1(b) n=19, m=6, $l_c=114$, $v_c=96$, i=0, $l_b=52$, $l_i=31$, l=83, v=65, $v_i=13$. In fig1(c) n=33, m=4, $l_c=132$, $v_c=100$, i=3, $\sum_{u=1}^{x} n_u\times d_u=3$, $l_b=62$, $l_i=35$, l=97, v=62, $v_i=3$.

V. Conclusions

The goal of our work is to find out the numerical relationships among the number of geometric structures of the FEM grid and its elements. This task is rather difficult and interesting in 3-D case and the results can be used to verify FEM grid directly especially for automatic FEM grid generation.

On the other hand, during the analysis topological relationship between FEM elements in a FEM grid is studied thoroughly. Thus many further researches and application can be performed. In the end an unified description of FEM procedure based on these numerical relationships can be gained, and this is especially effective for large FEM software programming.

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